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Thermal Conductivity of Poor Conductors

THERMAL CONDUCTIVITY OF POOR CONDUCTORS

BY

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I INTRODUCTION

A. Purpose of Investigation.- The purpose of this investigation was two fold: first to determine the method of measuring the thermal conductivity which is most applicable to poor conductors, especially those that can be molded in any desired form, and second, to determine the conductivity of some poor conductors.

The thermal conductivity of a number of building materials, such as concrete, gypsium and brick, has not been determined with exactness, if at all. These materials come in a class which we ordinarily call poor conductors of heat. There has been some work done on the relative conductivities of such materials but very little on the absolute conductivities. It is desirable to determine the thermal conductivity of some materials over a large range of temperature and this must necessarily enter in when selecting a suitable method.

B. Mathematical Theory.- The mathematical theory of heat conduction was first given by Fourier early in the nineteenth century. He covered the theory so completely that very little has been added since except the application of his theory and the solution of special problems.

The general equation for the conduction of heat given by Fourier is,

$$h^2 \left\{ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right\} = \frac{\partial \theta}{\partial t} \qquad (1)$$

where θ represents the temperature at any point and $h^2 = \frac{k}{c\rho}$, k is the conductivity, c the specific heat, and ρ the density. The term h^2 is what was called the "thermometric" conductivity by Maxwell and the "diffusivity" by Kelvin. The latter term is the one commonly used at the present time.

The above equation has been applied in only a few of the simpler

cases to the determination of the conductivity k. Angström applied the equation to a bar which was heated and cooled alternately in the middle. With the exception of Angström's method, all the applications of equation (1) have been to heat the substance until the steady state has been reached. Then the temperature at any point is independent of the time. Equation (1) then becomes,

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} = 0. \tag{2}$$

For linear flow of heat such as an infinite plate bounded by two faces, one at a temperature θ_1 and the other at θ_2 we get,

$$\frac{d^2\theta}{dx^2} = 0 \tag{3}$$

The solution of which is

$$\theta = Ax + B. \tag{4}$$

If the plate is taken as parallel to the yz plane and the faces at distances of x_1 and x_2 from the origin, we get for boundary conditions,

$$\theta = \theta_1 \text{ at } x = x_1 \quad \text{and} \quad \theta = \theta_2 \text{ at } x = x_2,$$

eliminating A and B in equation (4),

$$\theta = \frac{(x_2\theta_1 - x_1\theta_2)}{x_2 - x_1} - \frac{(\theta_1 - \theta_2)x}{x_2 - x_1}. \tag{5}$$

The rate heat energy is transferred through unit cross section W is defined as

$$W = -k\frac{\partial\theta}{\partial x}. \tag{6}$$

Equation (5) becomes

$$W = \frac{k(\theta_1-\theta_2)}{x_2-x_1} = \frac{k(\theta_1-\theta_2)}{u} \tag{7}$$

where u is the thickness of the plate. If we consider a cross section area of the plate A and heat flows for a time t, the total heat transferred through the section A in the time t is,

$$H = k \frac{\theta_1 - \theta_2}{n} At. \qquad (8)$$

This is the equation which has been most applied to determine the conductivity k.

The above equation applies in the same way for the flow of heat in a rod, if it is insulated so no heat can enter or leave except at the ends.

In applying Fourier's equation to the radial flow of heat in a cylinder where the heat is applied along the axis of the cylinder and flows out we have a two dimensional flow of heat. After the steady state has been reached we have

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \qquad (9)$$

if the axis of the cylinder lies on the z axis. By the relation $r^2 = x^2 + y^2$ we can express the temperature θ as a function of a single variable r.

$$\frac{\partial r}{\partial x} = \frac{x}{r} \quad \text{and} \quad \frac{\partial r}{\partial y} = \frac{y}{r}$$

$$\frac{\partial \theta}{\partial x} = \frac{d\theta}{dr} \frac{\partial r}{\partial x} = \frac{d\theta}{dr} \frac{x}{r}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{d^2\theta}{dr^2} \frac{x^2}{r^2} + \frac{d\theta}{dr} \frac{1}{r} - \frac{d\theta}{dr} \frac{x^2}{r^3}$$

$$\frac{\partial^2 \theta}{\partial y^2} = \frac{d^2\theta}{dr^2} \frac{y^2}{r^2} + \frac{d\theta}{dr} \frac{1}{r} - \frac{d\theta}{dr} \frac{y^2}{r^3}$$

putting these values in (9) we get

$$\frac{d^2\theta}{dr^2} + \frac{d\theta}{dr} \frac{1}{r} = 0 \qquad (10)$$

and since

$$\frac{d(r\frac{d\theta}{dr})}{dr} = r \frac{d^2\theta}{dr^2} + \frac{d\theta}{dr}$$

we have

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} = \frac{1}{r} \frac{d(r\frac{d\theta}{dr})}{dr} = 0 \qquad (11)$$

integrating

$$\theta = A \log r + B.$$

For boundary conditions, take r_1 and r_2 as the internal and external radii of the boundary surfaces of the cylinder, our conditions then are,

$$\theta = \theta_1 \text{ at } r = r_1 \quad \text{and} \quad \theta = \theta_2 \text{ at } r = r_2$$

$$\theta = \frac{(\theta_1-\theta_2)\log r}{\log r_1 - \log r_2} + \frac{\theta_1 \log r_2 - \theta_2 \log r_1}{\log r_2 - \log r_1} . \tag{13}$$

Getting the rate of flow of heat W as before

$$W = k \frac{(\theta_1-\theta_2)}{r(\log r_1 - \log r_2)} \tag{14}$$

or the flow of heat out of the cylinder per unit length per second

$$H = 2\pi r W = 2\pi k \frac{\theta_1 - \theta_2}{\log r_2 - \log r_1} . \tag{15}$$

C. Difficulties in Measuring Heat Conduction.- In determining the conductivity of a substance experimentally, there are in general three quantities to be measured, the quantity of heat, the temperature difference and the distance over which the temperature difference is measured.

The relative accuracy with which these three quantities can be measured depends to a considerable extent on the method applied to determine the conductivity.

If the heat is measured by a calorimeter method, it is very difficult to measure it with the accuracy desired, due principally to the fact that we have no non-conductors of heat like we have for electricity. If the heat is applied by an electric current, the heat flowing can be obtained from the input of electrical energy and we thus have a much more accurate method of measuring the heat.

The temperature at different points in a substance can be measured quite accurately by having small holes in the substance and

placing thermocouples made of small wire in these holes. It is necessary to assume however that the holes in the substance do not appreciably change the direction of the flow of heat.

The measurement of the distance between the points where the temperature difference is measured is usually a large source of error. For it is not always certain that the thermojunctions are in the center of the hole when the temperature measurements are made. As the temperature measurements are usually made at points close together this introduces a large error in the conductivity.

II. METHODS APPLICABLE TO THE DETERMINATION OF THE THERMAL CONDUCTIVITY OF POOR CONDUCTORS

There are only two general methods that have been applied to measure the thermal conductivity of poor conductors. These are the plate method and the cylinder method.

The plate method as first used consisted of a plate of the substance, the conductivity of which was to be determined with a steam chamber on one side and a water chamber on the other. The rate heat flowed through the plate was determined from the rise in temperature of the water in a certain time. The temperatures were taken as that of the steam and water. The conductivity could then be calculated from equation (8)

$$k = \frac{Hu}{(\theta_1-\theta_2)At} .$$

The values obtained by Clement for copper in this way were later found to be only about one one-hundredth the value of the conductivity of copper. This was explained by the fact that there was a layer of gas near the plate on the steam side which was not the same temperature as the steam and likewise the layer of water next to the

plate on the opposite side which was not at the same temperature as the rest of the water. Peclet found that by stirring both the steam and the water this error could be eliminated somewhat, but not entirely. It was then found that if holes were drilled in the substance and thermometers inserted, the temperature gradient could be determined much more accurately.

In the more recent investigations the thermometers have been replaced by thermocouples and in most cases the heat has been generated by electric currents.

Niven was the first to use the substance in the form of a cylinder to determine the conductivity. The method consists in having the source of heat in the axis of the cylinder and measuring the temperatures inside the cylinder at different distances from the center. If the length of the cylinders is at least three times the diameter there will be a region in the middle of the cylinder where the heat will all flow out radially.

Both from a theoretical and experimental point of view this method seems to be the most satisfactory for the measurement of the conductivity of good conductors as well as poor conductors.

Clement and Egy used this method to determine the conductivity of fire clay at high temperatures. Angell has applied the same method to determine the conductivity of nickel over a large range of temperature. Norton used a similar method of determining the conductivity of concrete at high temperatures.

III EXPERIMENTAL

A. Plate Method.- An attempt was made to use the plate method to determine the thermal conductivity of concrete with the hope that it could be used with a temperature range up to perhaps 1000°C.

Two concrete plates 33 cm. in diameter and 5.6 cm. thick were made up of one part Portland cement and four parts sand. No gravel was used in these plates so they would be of more homogeneous structure, and the temperature fall across the plates could be more accurately determined.

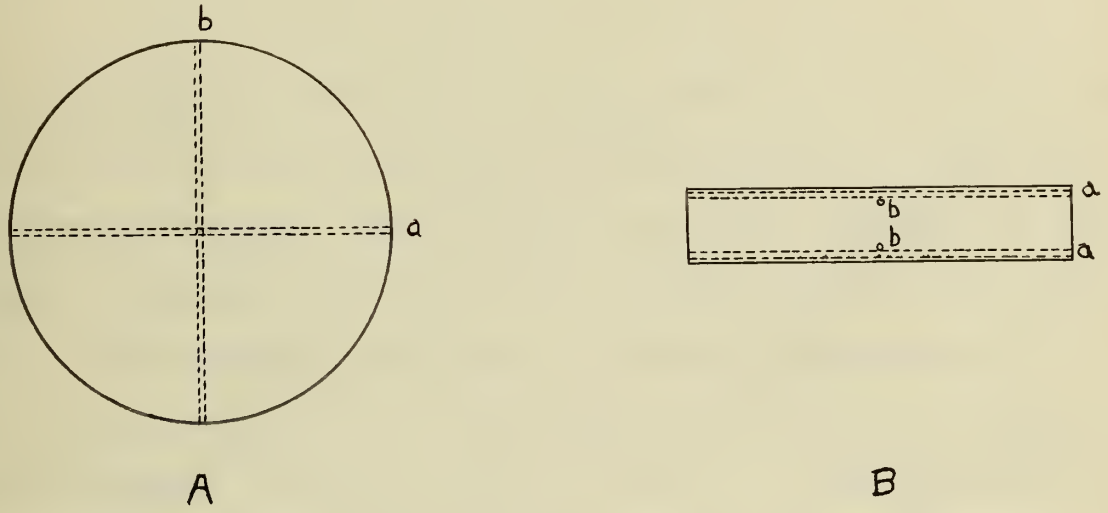


Fig. 1

In Fig. 1, A shows a horizontal section of one of the plates, and B a vertical section. The tubes a and b were small "Pyrex" glass tubes, which were embedded in the concrete. The thermocouples were placed in these tubes and the fall of temperature across the plate measured.

The heating coil was made of No.16 B and S gauge "Nichrome" wire. It was made in the form of a plate. The way in which the wire was wound is shown in Fig.3.

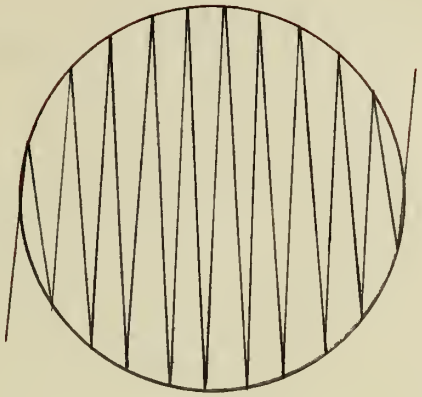


Fig. 2

This heating plate was made the same size as the concrete plates. The wire crossed the plate 63 times, which spaced the wires about one half a centimeter apart. The heating wires were embedded in "Alundum" cement purchased from Norton and Company, of Worcester, Massachusetts. This material was found to be quite satisfactory for embedding heating wires. It is mixed with water and can be applied to any desired shape. To bake it out a current is sent through the wire for several hours. The heating plate was about one centimeter thick and placed between the two concrete plates, so the heat flowed equally in both directions.

The diameter of the plates being nearly six times the thickness, it was thought that there would be a region of perhaps 6 or 8 centimeters in diameter in the center of the plates where the flow of heat would be linear and parallel to the axis of the plates. This was not found to be the case as is shown by Curves 1 and 2. The curves represent the fall of temperature across the plate in the direction perpendicular to the wires of the heating coil. The temperatures were measured at intervals of one centimeter across the plate. The

temperature fall in the direction parallel to the wires of the heating coil was practically the same. Curve 1 is for the hole nearest the heating plate and Curve 2 for the hole just above it and farther from the heating plate.

An attempt was then made to correct the flow of heat by sending heat in from the outside of the plate. To do this a coil of the same wire as used in the heating plate was wound around one of the plates as shown in Fig.3. These wires were also embedded in "Alundum".

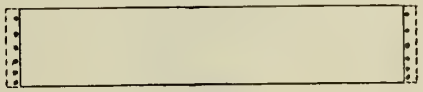
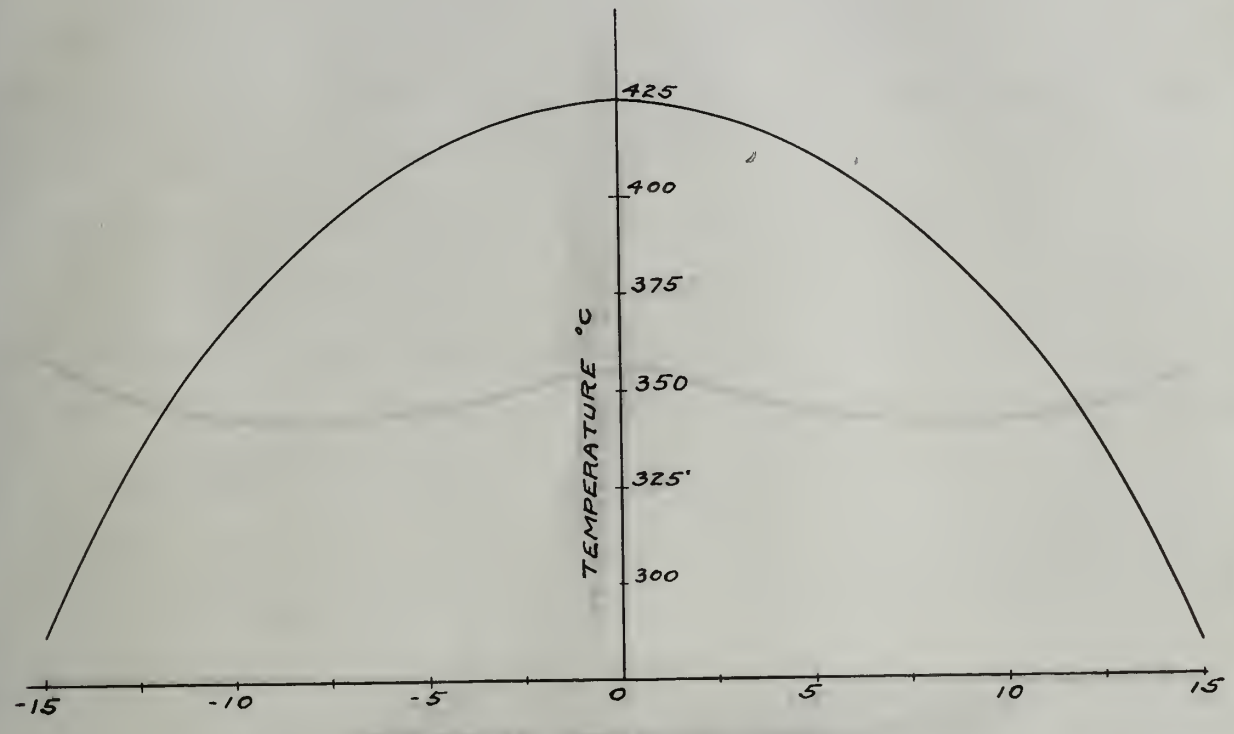


Fig. 3

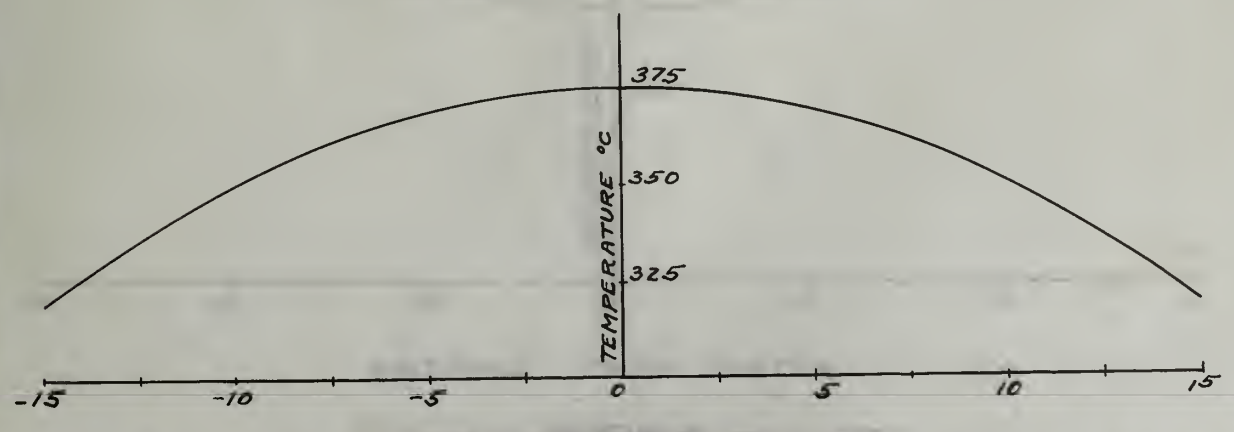
The wires were wound closer together at the lower side of the plate which was next to the heating coil. This was done so more heat would be sent in near the region of the higher temperature where the edge loss was greater.

The current through this outside coil was varied with respect to that through the heating plate until the smallest temperature difference across the middle of the plate was obtained. Curves 3 and 4 were taken the same way as 1 and 2 above. It can be seen that by this method the temperature difference across the plate was greatly reduced but there was not a region in the center where the temperature was constant. Curve 4 shows that too much heat was sent in near the upper side of the plate.

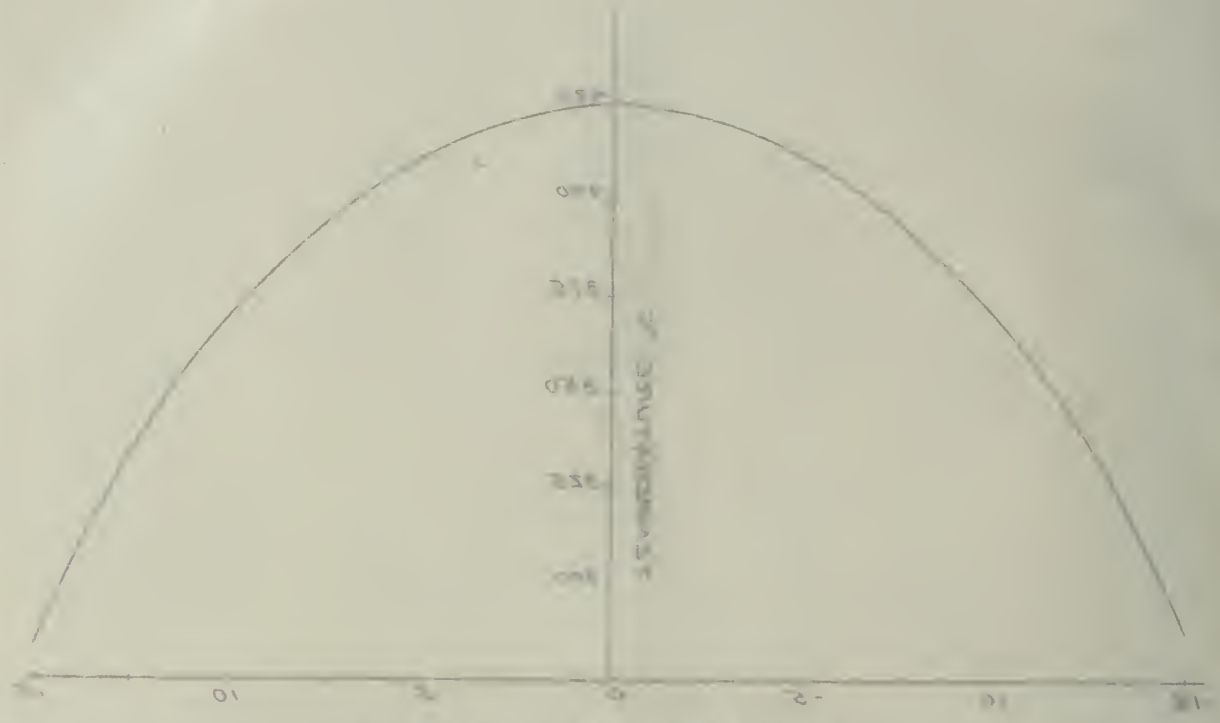
No conductivity measurements were carried out with this method as the flow of heat was not uniform enough that satisfactory results for the conductivity could be obtained. The results show that it

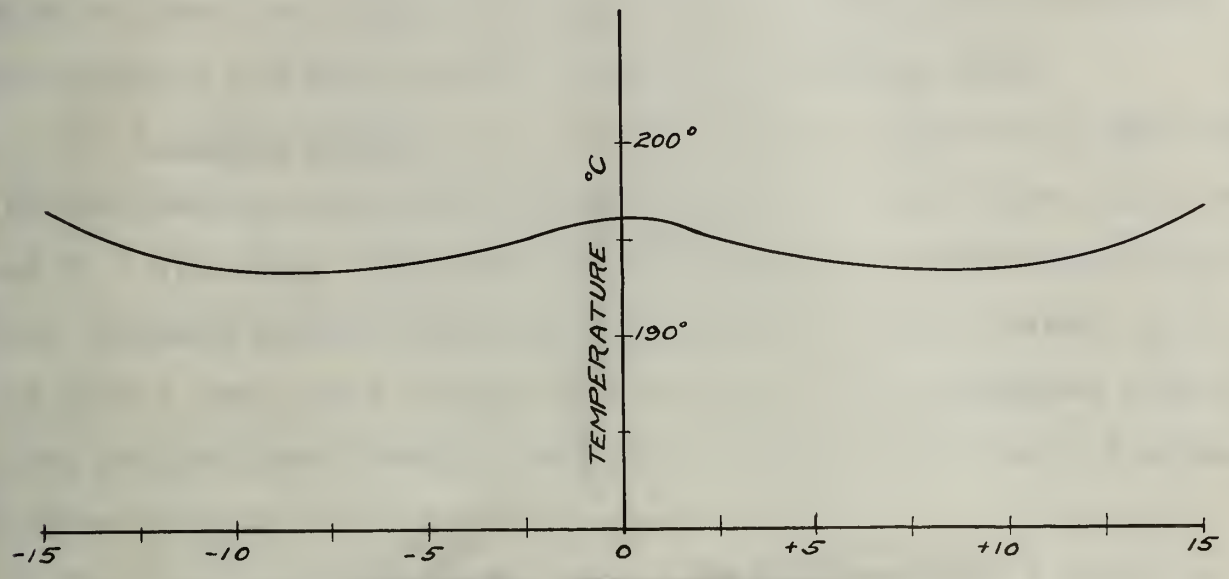


CURVE 1.

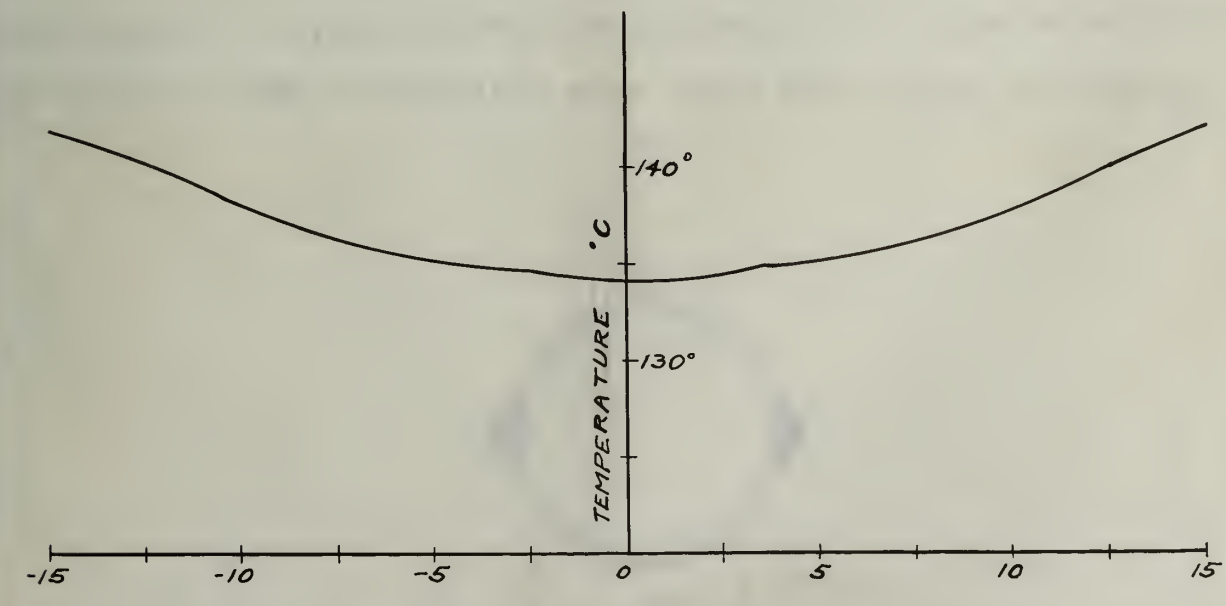


CURVE 2.

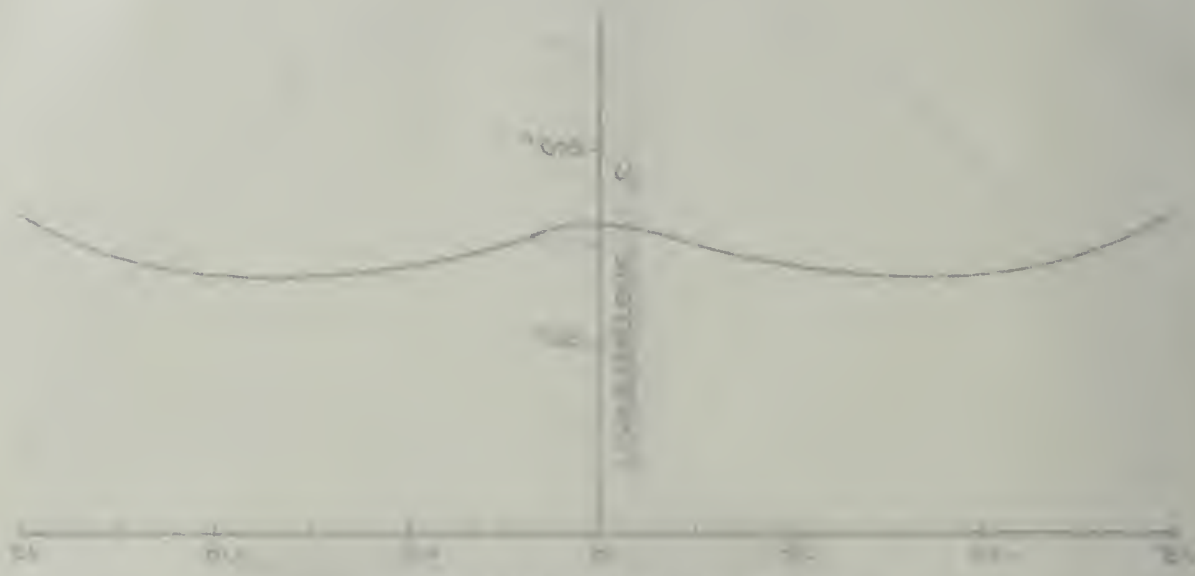




CURVE 3.



CURVE 4.



might be possible to correct for the heat loss at the edge of a plate by properly spacing the wire in the coil around the plate and adjusting the current through this coil. The objection to this method in determining the conductivity is that the spacing of the wire in the outside coil and the current through it would have to be determined experimentally for each current through the heating plate.

B. Cylinder Method.- In determining the conductivity by the cylinder method, the specimens used were cylinders 30 cm. in diameter and 61.5 cm. long. They were made of concrete, consisting of one part Portland cement, two parts sand and four parts gravel, by volume. The gravel used was a dolomitic limestone and the fineness modulus by sieve analysis was found to be 7.46. The sand used had a fineness modulus of 3.72.

The form used to mold the cylinders was made from a piece of wrought iron pipe 8 inches in diameter. The pipe was cut into lengthwise as shown by the section in Fig.4. Lugs were riveted on each half of the pipe and the two pieces bolted together as shown in the figure. Cast iron plates were fitted off to form the top and the

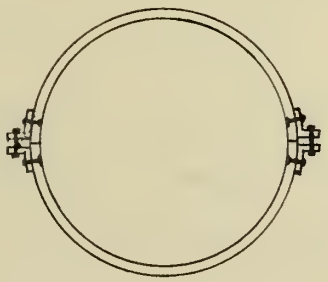


Fig. 4

bottom of the mold.

The cylinders were taken out of the form after having set for 48 hours. They were then wrapped in burlap, which was kept saturated with water for ten days. After that they were allowed to set in a dry atmosphere.

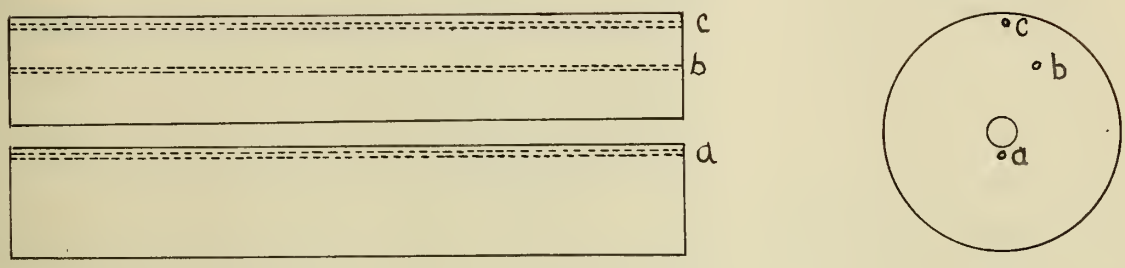


Fig. 5

Cross sections of specimen number 1 are shown in Fig.5. The hole through the center was 4 cm. in diameter. Thin walled "Pyrex" glass tubes about 3 mm. inside diameter were placed in the fresh concrete at different distances from the center and parallel to the axis of the cylinder. These are represented by a, b and c in Fig.5. The tube b was broken in tamping the concrete of specimen No.1 so only the temperatures at a and c could be measured.

The heating coil used with this cylinder was made by winding No.16 D and S gauge "Nichrome" wire on a common unglazed porcelain insulator tube and coating it over with "Alundum".

Test cylinder No.2 was made of the same kind of cement, sand and gravel. The proportions being the same as in No.1. Cross sections of this cylinder are shown in Fig.6. Instead of having the large

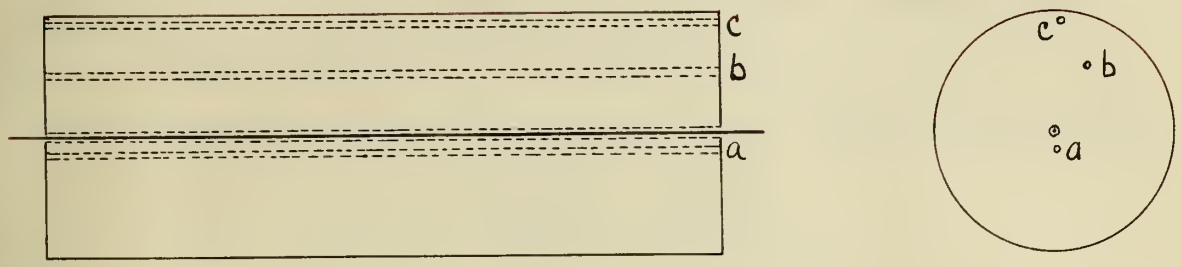


Fig.6

hole through the center, this cylinder had a straight piece of No.13 B and S gauge "Nichrome" wire inside a "Pyrex" glass in the center, as shown in Fig.6.

The conductivity measurements were made on each cylinder after the concrete had set for 83 days.

The heating current was taken from a motor-generator set equipped with a voltage regulator. The generator of this machine has a capacity of 300 amperes at 110 volts and gave a very constant source of electromotive force.

The temperatures were measured with copper constantan thermocouples calibrated at the boiling point of water and the melting points of tin and cadmium. The electromotive forces were measured with a "Wolff" potentiometer. A sensitive moving coil galvanometer made by the Weston Electrical Instrument Company was used with the potentiometer and by reading the galvanometer deflections, the electromotive force could be measured to one microvolt. The potential drop across the heating coil was measured with a Weston voltmeter. The heating current was measured with a Siemens and Halske ammeter.

After the temperature measurements were made, the cylinders were broken and the distances between the centers of the glass tubes and the axis of the cylinder were measured. These radii are represented by r_a, r_b and r_c in the table. The distances were measured in centimeters and the temperature in Centigrade degrees.

The conductivity was calculated, using equation (15) above.

$$k = H \frac{\log r_2 - \log r_1}{2\pi(\theta_1 - \theta_2)}$$

where

$$H = \frac{.239EI}{61.5} \text{ calories,}$$

or the heat flowing out unit length of the cylinder per second.

The results of the conductivity measurements are shown in Table I.

TABLE I

Specimen No.	r_a	r_b	r_c	E	I	θ_1	θ_2	θ diff.	k
1	3.53		8.89	42.00	4.40	96.34	67.86	28.48	.0051
1	3.53		8.89	72.3	7.12	244.60	138.04	116.63	.0035
1	3.53		8.89	90.03	8.71	360.13	176.43	183.7	.0034
2	1.64	6.14		4.88	31.67	68.46	55.19	13.27	.0036
2		6.14	8.87	4.88	31.67	54.59	50.81	3.78	.0063
2	1.64	6.84		6.64	30.4	113.22	83.74	33.48	.0056
2		6.14	8.87	6.64	30.4	83.79	75.73	8.06	.0052

The recorded values of k represent the quantity of heat in calories that flows in one second through a cross section area of one square centimeter and a thickness of one centimeter when the difference of temperature is one degree Centigrade.

Cylinder No.1 cracked in a number of places when heated to about 250° C, and this fact likely accounts for the discrepancy in the

values of the conductivity in the second and third rows. It was also found that there were some air spaces in the concrete of this cylinder. This would also lower the conductivity.

The only values for the conductivity of concrete which we have found are those given by Norton. The values he gives for the conductivity of gravel concrete of the same proportions as given above are, from .0021 at less than 100°C. to .0028 at 1100°C. He gives no information as to the nature of the gravel used or the time and condition of the concrete when tested. The nature of the gravel used as aggregate must have a considerable effect on the conductivity of concrete. For this reason no direct comparison can be made between the results given by Norton and those determined in these experiments.

More work is to be done on the conductivity of concrete, both over a large range of temperature and with different mixtures and aggregates.

IV CONCLUSIONS

1. It is possible to correct the flow of heat through a plate to a considerable extent by sending heat in from outside coils.

2. The cylinder method is the best method for the determination of the thermal conductivity of poor conductors.

3. The results indicate that the conductivity of 1-3-4 concrete is in the order of .0055 for temperatures of about 100°C.

4. The experiments carried out so far are not sufficient to give accurate values of the conductivity of concrete made with dolomitic limestone as the aggregate.

5. Further experiments should be carried out to determine the variation of the thermal conductivity with the temperature.

In conclusion I wish to express my appreciation to Professor A.P. Carman for the interest taken in this work and for his guidance and instruction in various phases of the investigation.

Photograph showing two cylinders after they were broken. Plates used in the experiments are also shown.

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